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SIG025Z	09/27/08	TCN	

9. PRIME CONTRACTOR	CODE	10. ADMINISTERED BY	CODE
Signal Innovations Group, Inc. 1009 Slater Rd., Ste. 200 Durham, NC 27703	3VKX5	DCMA Southern Virginia 190 Bernard Rd. BLDG 117 Fort Monroe, VA 23651	S5111A

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Fast Electromagnetic Solvers For Large-Scale Naval Scattering Problems

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Final Technical Report on ONR Contract N00014-06-C-0029

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4. TITLE AND SUBTITLE

Fast Electromagnetic Solvers For Large-Scale Naval Scattering Problems

5a. CONTRACT NUMBER

N00014-06-C-0029

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

Dr. Larry Carin

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Signal Innovations Group, Inc.
1009 Slater Rd., Ste. 200
Durham, NC 27703

8. PERFORMING ORGANIZATION REPORT NUMBER

SIG.ONR.017.FINAL

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Office of Naval Research
ONR 254: Emily McLaughlin (703) 696-7827
875 N. Randolph St.
Arlington, VA 22203-1995

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

A

13. SUPPLEMENTARY NOTES

14. ABSTRACT

Efficient modeling of electromagnetic scattering has always been an active topic in the field of computational electromagnetics. To reduce the memory and CPU time in the method of moments (MoM) solution, an efficient method based on pseudo skeleton approximation is presented in this report. The algorithm is purely algebraic, and therefore its performance is not associated with the kernel functions in the integral equations. The algorithm starts with a multilevel partitioning of the computational domain, which is very similar to the technique employed in multilevel fast multipole algorithm (MLFMA).Any of the impedance sub-matrices (with size of m£n) associated with the well-separated partitioning clusters (far interaction terms) is represented by the product of two much smaller matrices (with sizes of m£r and r£n), where r is the effective rank. Therefore, the memory requirement will be relieved and the total CPU time will be reduced significantly as well, since the rank is much smaller than the original matrix dimensions. It should be noted that we don't have to calculate all the impedance entries to implement

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16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
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Abstract

Efficient modeling of electromagnetic scattering has always been an active topic in the field of computational electromagnetics. To reduce the memory and CPU time in the method of moments (MoM) solution, an efficient method based on pseudo skeleton approximation is presented in this report. The algorithm is purely algebraic, and therefore its performance is not associated with the kernel functions in the integral equations. The algorithm starts with a multilevel partitioning of the computational domain, which is very similar to the technique employed in multilevel fast multipole algorithm (MLFMA). Any of the impedance sub-matrices (with size of $m \times n$) associated with the well-separated partitioning clusters (far interaction terms) is represented by the product of two much smaller matrices (with sizes of $m \times r$ and $r \times n$), where r is the effective rank. Therefore, the memory requirement will be relieved and the total CPU time will be reduced significantly as well, since the rank is much smaller than the original matrix dimensions. It should be noted that we don't have to calculate all the impedance entries to implement the aforementioned decomposition. Instead, we only need to calculate a few randomly chosen rows and columns of those impedance entries. Further compressions based on singular value decomposition (SVD) are performed so that the rank reaches its optimal limit, which leads to the optimized final matrix compression. Numerical examples are provided to show the validity of the new algorithm. Future work directions are also discussed in this report.

Index Terms

Pseudo skeleton approximation, adaptive cross approximation, low rank matrix approximation, singular value decomposition, electromagnetic scattering, sea surface scattering

I. Introduction

Method of moments (MoM) [1] has been a very popular approach in solving electromagnetic scattering problems. However, it has also raised challenging issues since it suffers from the memory requirement for a large dense impedance matrix and computational complexity for large-scale problems. It is well known that significant progress has been made in employing the fast multipole method (FMM) [2], [3], [4], [5], to relieve the aforementioned limitations. For example, multilevel fast multipole algorithm (MLFMA) [6], [7] incorporated with the iterative techniques can reduce the numerical complexity to $O(N \log N)$ to solve integral equations. However, one of the major disadvantages of this approach is that the algorithm is NOT independent of the integral equation kernel. That is, for integral equations with different kernels, one has to make appropriate modifications [7] to implement the fast algorithm. Another approach to the compression of operators is based on wavelets [8], [9], which exploit the smoothness of the elements of the matrix viewed as a function of their indices and tends to fail for highly oscillatory operators.

On the other hand, approaches based on low-rank representation of impedance matrix blocks have been also introduced in the field of computational electromagnetics. These include the so-called IES^3 [10], IE-QR [11], [12], PILOT (predetermined interaction list oct-tree) [13], and ACA (adaptive cross approximation) algorithms [14], [15]. In these methods, all the impedance matrix blocks (assume that the size of the matrix is $m \times n$) associated with the far interaction terms are represented by a product of two much smaller matrices with sizes of $m \times r$ and $r \times n$, where r is the effective rank of the matrix which can be determined numerically. Thus, the memory requirement can be reduced from $m \times n$ to $r \times (m+n)$, and the same ratio of CPU time saving can be obtained for matrix-vector product. The beauty of these algorithms is their purely algebraic nature. That is, the computational speed-up is achieved by employing linear algebra manipulations of the impedance matrix. Thus, the implementations of these algorithms do not depend on the complete knowledge of the integral equation kernels. However, the algorithms' complexity is $O(r^2(m+n))$, which is not trivial when m or n is big enough. And the other fatal limitation of these algorithms is that they are not stable when the rank of the matrix is big according to our experience.

It should be noted that randomized algorithms [16], [17]can also be used for the construction of low-rank approximation to matrices. The idea is that a randomized matrix is employed to project the low-rank matrix to a much smaller space. Thus, a new matrix with much smaller dimensions is obtained. Then any low-rank decompositon technique can be used to get the appropriate basis functions. And the corresponding coefficient matrix can be calculated straight forward once the basis functions are known. Unfortunately, all the entries of the low-rank matrix are needed for this method.

In this report, the pseudo skeleton approximation method [18] is employed for the purpose of matrix decomposition. There are several advantages of the algorithm compared with the aforementioned algorithms. First, the algorithm is very stable, which is very important; Secondly, its computational complexity is $O(r^3)$, which is independent of m or n.

The report is organized as follows: First in Section II the pseudo skeleton approximation algorithm is introduced, and a technique based on singular value decomposition (SVD) [19] is employed for further matrix compression. Then some numerical results are presented in Section III to demonstrate the performance of the current approach, followed by the discussions and future work directions.

II. Efficient Low-Rank Matrix Decomposition Using Pseudo Skeleton Approximation

In the field of computational electromagnetics, and many other areas, one often encounters matrices whose ranks are much lower than their dimensionalities (rank deficient). Discretization of integral equations almost always results in matrices of this type. For such kind of matrices, one is tempted to "compress" the matrices in question so that they could be efficiently applied to arbitrary vectors, and at the same time the storage requirement can also be reduced (compressed) as well.

It should be noted that the entire impedance matrix obtained through MoM is neither singular nor rank deficient except at the internal resonances. However, if all the unknowns are grouped into clusters like in MLFMA, then all the sub-matrix blocks representing the interactions between two well-separated clusters

(associated with the far interaction terms) are rank deficient and they can be compressed efficiently by using the pseudo skeleton approximation method described below. As in MLFMA, all the diagonal sub-matrix blocks (associated with self interactions) as well as all the sub-matrix blocks associated with the interactions of any neighboring clusters are to be calculated directly via the conventional MoM approach.

In this Section, we will first give a brief introduction of the low-rank approximation. Then details of the pseudo skeleton approximation will be presented.

A. *Outline of low-rank matrix compression*

Without loss of generality, assume that the size of a low-rank matrix A (representing interactions between two well-separated clusters) is $m \times n$. Our main aim is to decompose the matrix A into two much smaller sub-matrices U and V. The original matrix A can be reconstructed through a product of U and V. Namely,

$$A(m \times n) \approx U(m \times r)V(r \times n) \tag{1}$$

where r is the effective rank of matrix A.

Instead of storing $m \times n$ impedance entries in the conventional method, the above low-rank compression technique only requires to store $r \times (m + n)$ impedance entries.

Similarly, when the matrix A is directly applied to a vector, the computational complexity is $m \times n$. In contrast, we can first apply matrices V and then U to the vector in sequence, the associated complexity will be $r \times (m + n)$.

For the electromagnetic problems, generally $r \ll \min(m, n)$. Therefore, much CPU time will be reduced and the memory requirement will be relieved significantly as well.

It has been proofed theoretically that SVD would find the best decomposition with given rank. In other words, for a given accuracy, SVD will find the associated lowest rank. However, the algorithm is very expensive, especially when the matrix dimensions are big. Direct application of QR factorization has similar drawback. Therefore, in this report pseudo skeleton approximation will be employed to decompose matrix A into U and V.

B. *Pseudo skeleton approximation*

Before going to the details of the pseudo skeleton approximation, we first review the skeleton approximation method.

Assume that the rank of the matrix A is r. Then there exists a nonsingular $r \times r$ submatrix $\hat{A}$ in A. Denote the columns and rows of A containing the submatrix $\hat{A}$ by C (with size of $m \times r$) and R (with size of $r \times n$), respectively. That is, submatrix $\hat{A}$ is the intersection of C and R. Then it is easy to verify that

$$A(m \times n) \approx C(m \times r)\hat{A}^{-1}(r \times r)R(r \times n) \tag{2}$$

This decomposition is known as a skeleton approximation of A.

The problem with skeleton approximation is that one should identify which columns and rows should be chosen. Random selection will lead to a singular $\hat{A}$, thus not enough bases are embedded in those columns and rows, and the inverse will not be available, which results in a fail decomposition.

ACA can be applied to get the above decomposition adaptively. That is, columns of C and rows of R are iteratively added until an error criterion is reached. However, as mentioned before, ACA is still expensive in the sense of complexity, and it is not stable when the rank of matrix is big enough.

Actually, the matrix can be approximated by

$$A(m \times n) \approx C(m \times r)G(r \times r)R(r \times n) \tag{3}$$

where G is not necessary equal to the inverse of $\hat{A}$ and even not necessarily nonsingular. For example, G can be chosen as the pseudo inverse of $\hat{A}$. This kind of decomposition is called the pseudo skeleton approximation.

Once the matrices C, G, and R are obtained, one can easily obtained the matrices U and V defined in (1):

$$U = C \tag{4}$$

$$V = GR \tag{5}$$

Or we can have:

$$U = CG \tag{6}$$

$$V = R \tag{7}$$

Unlike using an iterative approach in ACA to find the columns of C and rows of R, here we just randomly choose k columns and k rows from A, where k is a number large enough so that r most important bases will be embedded in both column and row sub-matrices. Numerical experiments show that $k = 3r$ is good enough to obtain excellent results.

The complexity of computing the pseudo inverse of $\hat{A}$ is $O(r^3)$, which is much less compared to ACA considering the fact that $r \ll \min(m, n)$.

C. Further compression of the matrices

It should be noted that the dimensions of the matrices C, R, and G are $m \times k$, $k \times n$, and $k \times k$, respectively. This guarantees that enough information is included in the matrices C and R. Therefore, the sizes of the associated matrices U and V are $m \times k$ and $k \times n$, respectively.

U and V must contain much redundancies since the rank (r) of the matrix A is smaller than k. On the other hand, the vectors inside U and V are generally not orthogonal. To remove the redundancies, both QR factorization and SVD can be employed here.

First employing QR factorization to decompose the matrix U and the adjoint of V, we have:

$$U(m \times k) = Q_u(m \times k)R_u(k \times k) \tag{8}$$

$$V'(n \times k) = Q_v(n \times k)R_v(k \times k) \tag{9}$$

Then SVD is employed to decompose the product of matrices $R_u(k \times k)$ and $R'_v(k \times k)$:

$$\hat{U}(k \times r)\hat{S}(r \times r)\hat{V}(r \times k) = R_u R'_V(k \times k) \tag{10}$$

During this step, the effective rank r of the matrix A is determined:

$$r = sum(|diag(\hat{S})| > tol \cdot |\hat{S}(1,1)|) \tag{11}$$

where tol is the relative tolerance. Generally, tol is chosen to be 10^{-3}.

Hence the final version of the decomposition of matrix A is as follows:

$$U_{final}(m \times r) = Q_u(m \times k)\hat{U}(k \times r) \tag{12}$$

$$V_{final}(r \times n) = \hat{S}(r \times r)(Q_V(n \times k)\hat{V}'(k \times r))' \tag{13}$$

Note that QR factorization and SVD are applied on matrices with much smaller dimensions.

III. Numerical Examples

In this Section, we will present some numerical examples to show the accuracy and efficiency of the new algorithm.

A. Selection of the sample numbers

An EM related impedance submatrix representing interaction between well separated groups is employed here for the study of the selection of the sample numbers.

The size of the impedance matrix is 280×280, and its effective rank is 8 (determined numerically according to equation 11).

The relative errors (the ratio of the Frobenius norm of the difference matrix to the original matrix) as a function of sample numbers are shown in Figure 1.

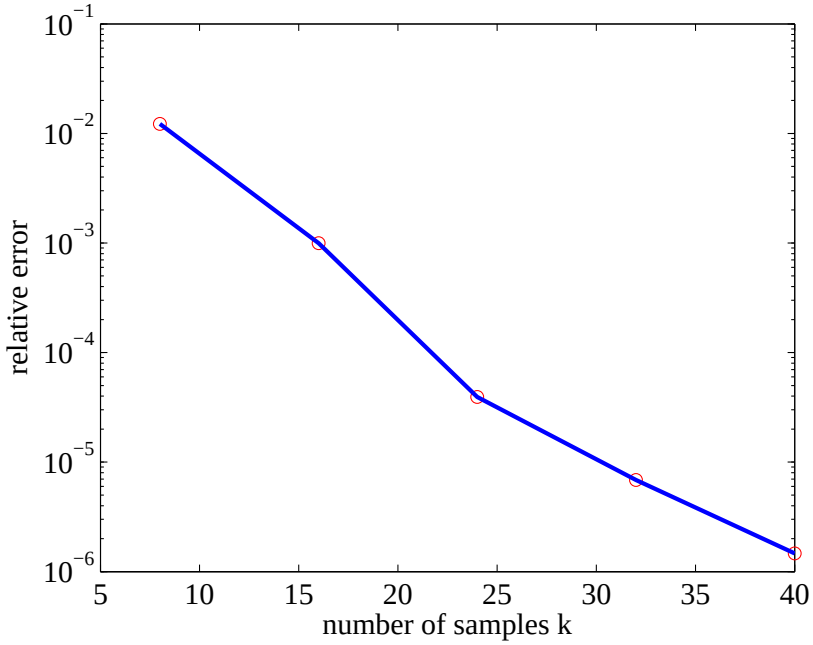


Fig. 1. Relative errors as a function of sample numbers. The horizontal axis is the number of samples, and the vertical axis is the relative error.

From the Figure, we can see that the relative error is in the order of 10^{-5} when the number of samples is 3 times of the effective rank. The relative error is in the order of 10^{-4} when the number of samples is just twice of the effective rank, this should be good enough for most applications.

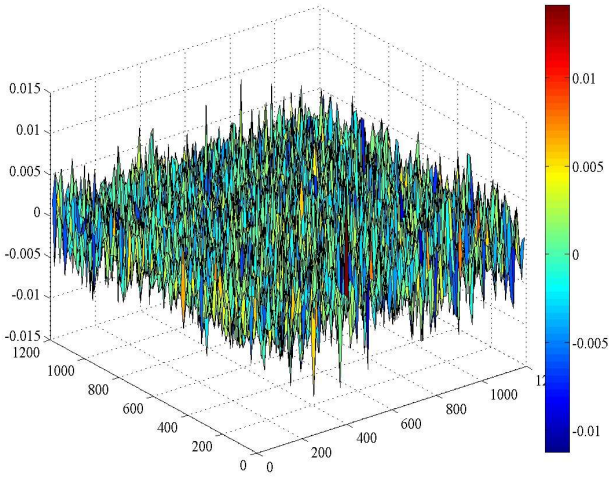
B. Accuracy of the pseudo skeleton approximation

To test the accuracy of the algorithm, we generate a random complex low-rank matrix. The size of the matrix is 1200×1200, and its rank is 10. The real and imaginary part of the matrix are shown in Figure 2.

30 randomly chosen columns and rows are used to obtain its pseudo skeleton decomposition. The differences between the reconstructed matrix and the original one are shown in Figure 3. Clearly, we can see that the pseudo skeleton approximation algorithm performs excellent.

C. RCS of a rectangular PEC plate

When the pseudo skeleton approximation method is applied for electromagnetic scattering problem, as mentioned before, the computational domain is first broken into a lot of subgroups at different levels like



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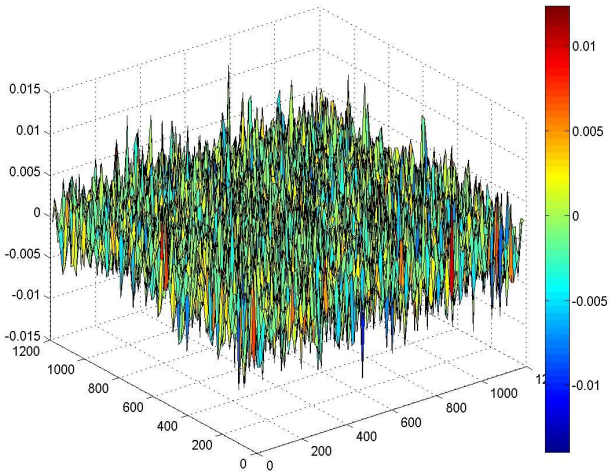

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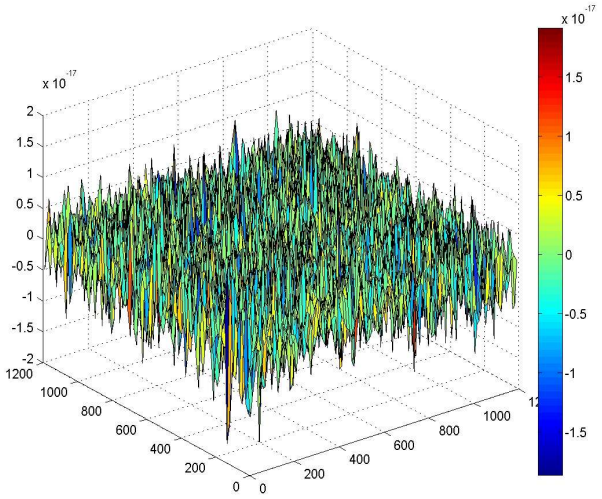
Fig. 2. A random complex low-rank matrix. The size and rank of the matrix are 1200×1200 and 10, respectively. (a)Real part; (b)Imaginary part.



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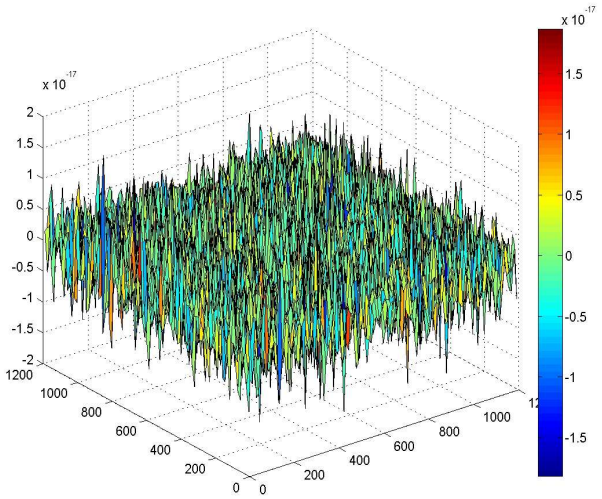

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Fig. 3. Difference between the reconstructed matrix and the original one. (a)Real part; (b)Imaginary part.

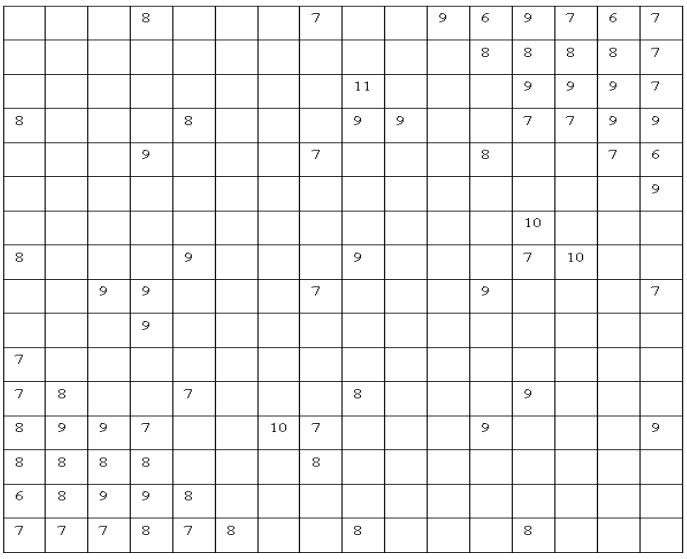


Fig. 4. Typical rank map of an EM problem. The target here is a $8\lambda \times 8\lambda$ rectangular plate, and the computational domain is divided into 16 subgroups (2 levels). The sizes of all the submatrices are around 1200×1200.

in MLFMA. In each level, those submatrices representing the interactions of well-separated subgroups are compressed via the pseudo skeleton approximation. Then the equations are solved by the conjugate gradient method.

As an example, a typical rank map of a rectangular plate is shown in Figure 4, where the computational domain is broken in 2 levels. From the Figure we can see that there are a lot of low-rank submatrices, and the ranks are very small compared to the their dimensions (around 1200×1200). And those submatrices associated with the neighboring subgroups (blank blocks in Figure 4 can be further divided in higher levels. Therefore, a lot of memory and CPU time can be saved.

To show the performance of the pseudo skeleton approximation, a rectangular PEC plate is considered here for the electromagnetic scattering analysis. The size of the plate is $4\lambda_0 \times 4\lambda_0$, where λ_0 is the wavelength in free space. The incident angles are: $\theta_i = 45^0$, and $\phi_i = 0$; and the scattering angles are: $\theta_s = 45^0$, and ϕ_s varies from $0 \sim 360^0$. The associated radar cross sections (RCS) are shown in Figure 5.

MoM results are also presented as reference solutions. Four different curves are shown in each Figure. We can observe that the low-rank approximation results of both co- and cross-polarizations agree with the MoM solutions very well.

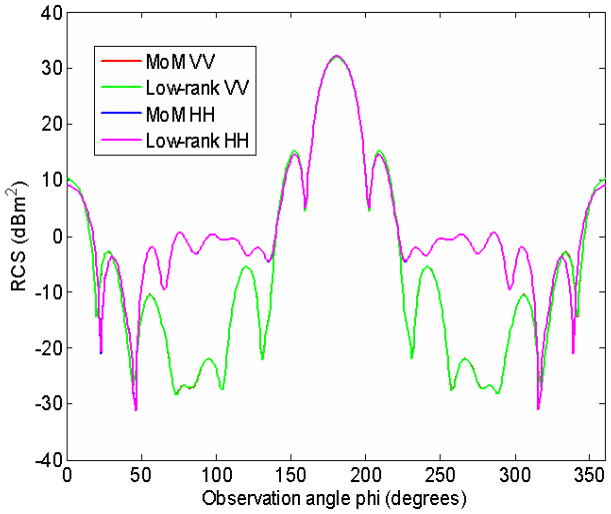
D. RCS of rough sea surface

The electromagnetic scattering analysis of an example rough sea surface is presented in this Section. The sea surface used in this example is based on the Pierson-Moskowitz model. The schematic of the sea surface generation is shown in Figure 6.

The Pierson-Moskowitz spectrum is defined as follows:

$$S(\omega) = \frac{\alpha g^2}{\omega^5} \exp[-\beta(\frac{\omega_0}{\omega}^4)] \tag{14}$$

where $\omega = 2\pi f$, $\alpha = 8.1 \times 10^{-3}$, $\beta = 0.74$, $\omega_0 = g/U_{19.5}$. f is the frequency of the electromagnetic wave, $U_{19.5}$ is the wind speed at a height of 19.5m above the sea surface, and g is the acceleration of gravity. An example rough sea surface with rms height of 0.1m (equivalent wind speed is 4.33m/s) is shown in Figure 7. In this example, the frequency of the incident wave is $300MHz$, hence the roughness



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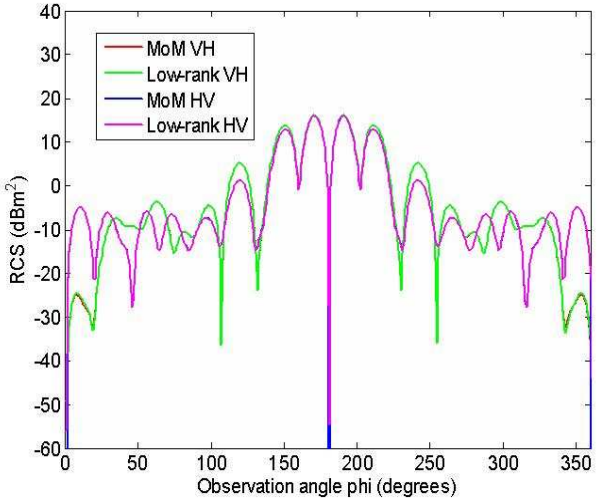

(b)

Fig. 5. RCS of a rectangular PEC plate: MoM vs. pseudo skeleton low-rank decomposition. (a)Co-polarization; (b)Cross-polarization. The horizontal axis is the observation angles, and the vertical axis is the radar cross section

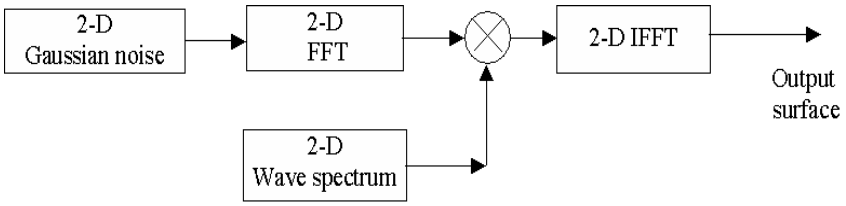

Fig. 6. Schematic of rough sea surface generation.

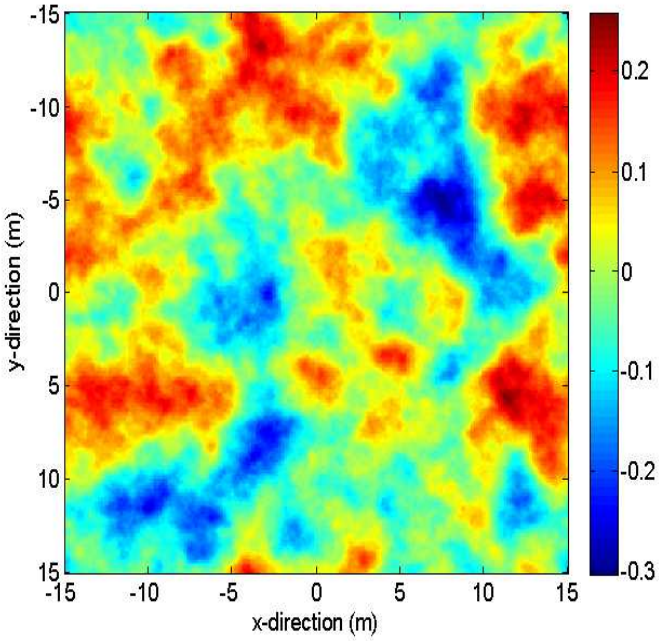

Fig. 7. An example rough sea surface with rms height of 0.1m.

of the sea surface is quite small in wavelength. Therefore, the reflection around the forward direction is still very strong, as expected.

The size of the sea surface is $10\lambda_0 \times 10\lambda_0$. The incident elevation angles are $\theta_i = -75^0$ and $\phi_i = 0$. To reduce the edge effects, a tapering function is employed for smoothing. The bistatic RCS of 1 realization is shown in Figure 8.

E. Complexity of the algorithm compared with MLFMA

To show the performance of the pseudo skeleton approximation, here the CPU time for calculate one matrix-vector product is compared with the counterpart of MLFMA.

Five cases with different number of unknowns are considered by both MLFMA and pseudo skeleton approximation, and the CPU times are shown in Figure 9. As a reference, a line proportional to $N \log N$ is also presented. From the Figure, one can observe that the algorithm based on the pseudo skeleton approximation is about 10 times faster than MLFMA.

It should be noted that there is a overhead to compress all the submatrices associated with the well separated groups. However, one often need to calculate monostatic RCS at a lot of different angles in

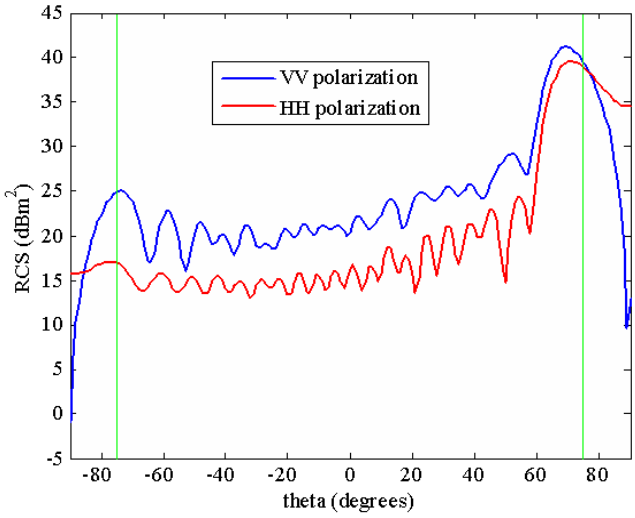

Fig. 8. Bistatic RCS of rough sea surface. The horizontal axis is the angle θ_s, and the vertical axis is the RCS.

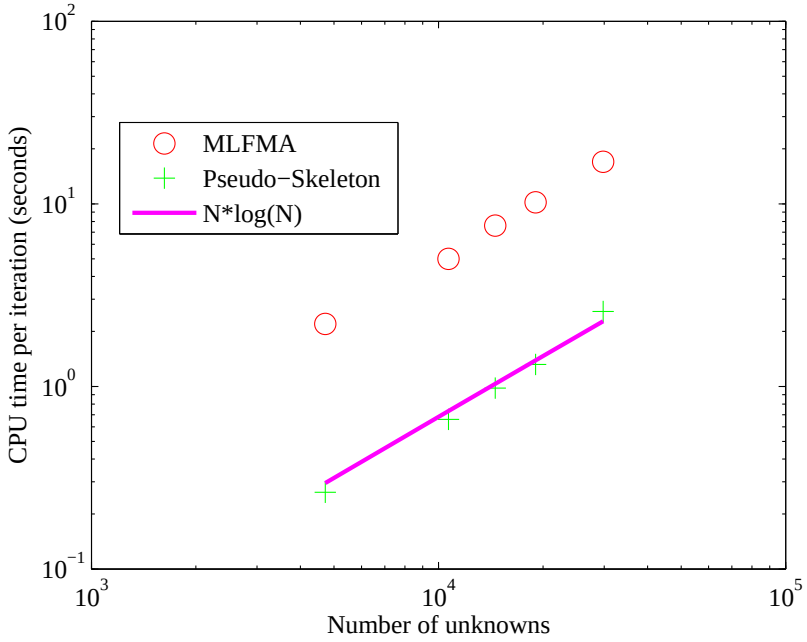

Fig. 9. Comparison of CPU time between MLFMA and the pseudo skeleton approximation. The horizontal axis is the number of unknowns, and the vertical axis is the CPU time in seconds.

reality. That is, there will be a large number of right-hand sides in solving the Maxwell integral equations, while the impedance matrix (the left-hand side) remains unchanged. In these cases, the algorithm based on the pseudo skeleton approximation should perform much better since the overhead will be amortized.

IV. Discussions and Future Work

In this report, we present a novel algorithm based on pseudo-skeleton approximation for fast electromagnetic scattering analysis. In summary, the algorithm is purely algebraic in nature and hence its

implementation is integral equation kernel independent. The algorithm first divides the whole computational domain into a lot of groups in different levels, exactly the same as wha is done in MLFMA. Then all the submatrices representing well separated groups are compressed by the pseudo skeleton approximation. It should be noted that only partial impedance entries are needed for this step, hence the compression can be implemented efficiently. Next, further compressiones based on SVD are employed so that the sizes of the decomposed submatrices will be close to their real ranks. Thus, the compressions are maximized. Numerical examples show that the method does perform very well compared with the conventional MLFMA.

There are a lot of work for further exploration in this area. An imediate list is as follows:

(1)Further optimize the algorithm, so the memory requirement and the CPU time can be further reduced. For example, check if two neighboring rank defficient submatrices can be merged. Without loss of generality, we consider two neighboring submatrices with size of 1000×1000 and rank of 10 (this is a typical case in EM problems). Based on the pseudo skeleton approximation, both of them can be compressed as a produce of two smaller submatrices with sizes of 1000×10 and 10×1000. The total memory requirement should be $2 \times (1000 \times 10 + 10 \times 1000) = 40000$ for them. However, if we can integrate them together, and assume they are neighbored along column direction and the rank is still 10, then the total memory requirement will be $1000 \times 10 + 10 \times 2000 = 30000$, which leads to a 25% saving in both memory and CPU time.

(2)When very large targets, for example, very large area of rough sea surfaces, are considered, we may incoporate the overlapped domain decomposition method (ODDM)[**?**] with the pseudo skeleton approximation. ODDM is a improved version of the forware/backward buffer region sweep method for 3D problems. The whole domain is divided into a few overlapped subdomains. The pseudo skeleton approximation is used for each subdomains. Once the current distributions of all the overlapped subdomains are obtained, an iterative procedure can then be employed to get the current distribution on the whole domain. Since the spurious edge effect of each subdomain can be effectively reduced becasue of the overlapped regions, the convergence of the ODDM iterative process can be very fast (It was shown that the iterative process converges in 2 or 3 iterations [20], [21]).

Since each subdomain can be modeled separately in sequence, thus the memory requirement is actually mainly determined by the size of each subdomain, not the whole computational domain. On the other hand, the current distribution on each subdomain can even be saved on hard disk, and be read later for the iterative purpose. Since the iterative process generally converges in 2 or 3 interations, the time for reading information from hard disk is definitely acceptable. Thus large targets can be easily handled without much difficulties.

(3)The algorithm based on the pseudo skeleton approximation is perfectly suitable for parallelization. All the subgroups can be easily assigned to different processors, and all the associated submatrices can be decomposed locally. More important is that no information is needed to be exchanged among the processors during the compression step. Similarly, the decomposition submatrices U and V can be allocated to memory locally as well. Thus the parallelized version of pseudo skeleton approximation method could be very efficient, compared to the parallelized MLFMA, where a lot of information is passed around all the processors.

Combinding the parallelization technique, ODDM, and the pseudo skeleton approximation, it should be very promising to solve very large problems.

(4)After parallelizing the algorithm, larger targets will be manageable. On the basis of that, we can further investigate the submatrices representing the interactions between very well-separated groups. The contributions of those submatrices should be very small and therefore they can be neglected and there is no need to do the compressions. Once again, a lot of memory can be saved.

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